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Generalized parton correlation functions for a spin-0 hadron

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Abstract: The fully unintegrated, off-diagonal quark-quark correlator for a spin-0 hadron is parameterized in terms of so-called generalized parton correlation functions. Such objects are of relevance for the phenomenology of certain hard exclusive reactions. In particular, they can be considered as *mother distributions* of generalized parton distributions on the one hand and transverse momentum dependent parton distributions on the other. Therefore, our study provides new, model-independent insights into the recently proposed nontrivial relations between generalized and transverse momentum dependent parton distributions. As a by-product we obtain the first complete classification of generalized parton distributions beyond leading twist.

Keywords: Deep Inelastic Scattering, Hadronic Colliders, Spin and Polarization Effects, Parton Model.

Contents

1. Introduction

Exploring the partonic substructure of hadrons by means of hard scattering processes has a long history. Perhaps the most important reaction in this context is inclusive deep inelastic lepton-nucleon scattering providing information on the (ordinary) quark and gluon parton distributions (PDFs) of the nucleon. In the meantime an enormous amount of knowledge about the PDFs has been collected, especially in the unpolarized case.

Nevertheless, the information contained in PDFs is limited to one dimension in the sense that PDFs merely tell us how the parton momenta parallel to the hadron momentum are distributed. A more complete picture of the parton structure of hadrons is encoded in two other types of distributions which are currently subject to intense studies. The generalized parton distributions (GPDs) [1] constitute one such type. They can be measured in hard exclusive reactions such as deep virtual Compton scattering or hard exclusive meson production (for reviews see, e.g., refs. [2 − 5]). In particular, when being transformed to the impact parameter space, GPDs contain information about the spatial distribution of partons in a plane perpendicular to the hadron momentum [6 − 9]. The second kind of generalized functions, the transverse momentum dependent parton distributions (TMDs), not only depend on the longitudinal but also on the transverse motion of partons inside a hadron. The TMDs enter the QCD-description of hard semi-inclusive reactions like semi-inclusive deep inelastic scattering (SIDIS) or the Drell-Yan (DY) process (see, e.g., refs. [10, 11] and the review articles [12, 13]).

The GPDs and TMDs, *a priori*, are considered as independent functions. However, recently nontrivial relations between these two classes of objects have been suggested in the literature [14 – 20]. Of particular interest are the relations between GPDs and the so-called (naïve) time-reversal odd (T-odd) TMDs like the Sivers function [21, 22] and the Boer-Mulders function [23], because they provide an intuitive connection between transverse single spin asymmetries observed in different hard semi-inclusive processes on the one hand and the distortion of parton distributions on the other. Although many nontrivial relations between GPDs and TMDs were established in simple spectator models (see [20] for an overview), no model-independent relations have been obtained so far.

The purpose of the present paper now is to investigate the structure of the generalized (off-diagonal) quark-quark correlator for a hadron, where for simplicity the analysis is restricted to a spin-0 hadron. This correlator is parameterized in terms of objects which we call generalized parton correlation functions (GPCFs). They depend on the full 4-momentum of the quark and, in addition, contain information on the momentum transfer to the hadron. Both the GPDs as well as the TMDs appear as two different limiting cases of the GPCFs. In other words, this means that the GPCFs can be considered as *mother distributions* of GPDs and TMDs [24, 25, 4]. The GPCFs also have a direct connection to the so-called Wigner distributions — the quantum mechanical analogues of classical phase space distributions — of the hadron-parton system [24, 25, 4].

Our motivation for carrying out this study is essentially fourfold: first, the GPCFs are the most general two-parton correlation functions of hadrons. As such they contain a maximum amount of information about the partonic structure of hadrons. Despite this fact no classification of these objects has been provided in the literature so far. Second, we want to find out which of the GPDs and which of the TMDs arise from the same *mother distributions*. Third, we hope, in particular, to get new, model-independent insights into the above mentioned nontrivial relations between GPDs and TMDs. (For a first account on this topic see the conference contribution in ref. [26].) Fourth, in the case of hard exclusive reactions it is known that, for the kinematics of current experiments at COMPASS, HERMES, and Jefferson Lab, effects going beyond the collinear parton approximation can be quite important (see, e.g., refs. [27 – 29]), though in principle they often can be considered as subleading twist. (For a related discussion in connection with more inclusive processes see [30] and references therein.)

The plan of the manuscript is as follows. In the next section the parameterization of the generalized quark-quark correlator in terms of GPCFs is derived. The results obtained there form the basis for the rest of the paper. In section 3 we consider the so-called generalized transverse momentum dependent parton distributions (GTMDs), which arise when integrating the GPCFs upon one light-cone component of the quark momentum. The GTMDs are the objects that can directly enter the description of hard exclusive reactions. It is worth mentioning that GTMDs for gluons have already been exploited previously in order to compute diffractive vector meson [31] and Higgs production [32]. In such processes GTMDs appear even at leading order of a twist expansion. The TMD-limit and the GPD-limit for the GTMDs are investigated in section 4. This allows us to obtain the first complete counting of GPDs beyond leading twist. In particular, we also study which

GPDs on the one hand and TMDs on the other have the same *mother distributions*. On the basis of our results we are able to investigate the model-independent status of possible nontrivial relations between GPDs and TMDs. Section 5 contains the conclusions. The exact relations between the GPCFs and the GTMDs defined in the manuscript are given in appendix A, while in appendix B our model-independent study is supplemented by the calculation of the GTMDs in a simple model for a spin-0 hadron.

2. Generalized parton correlation functions

In this section we derive the structure of the generalized, fully-unintegrated quark-quark correlator for a spin-0 hadron which is defined as

$$W_{ij}(P,k,\Delta,N;\eta) = \int \frac{d^4z}{(2\pi)^4}\, e^{ik\cdot z}\, \langle p' \,|\, \bar{\psi}_j\Big(-\frac{1}{2}z\Big)\, \mathcal{W}\Big(-\frac{1}{2}z,\frac{1}{2}z\,\Big|\,n\Big)\, \psi_i\Big(\frac{1}{2}z\Big)\,|\,p\rangle\,. \quad (2.1)$$

The correlator W depends on the average momentum $P = (p+p')/2$ of the initial and final hadron, the momentum transfer $\Delta = p' - p$ to the hadron, and the average quark momentum k. (For the kinematics we also refer to figure 1.) The Wilson line $\mathcal{W}$ ensures the color gauge invariance of the correlator in eq. (2.1) and is running along the path

$$-\frac{1}{2}z \;\rightarrow\; -\frac{1}{2}z + \infty\cdot n \;\rightarrow\; \frac{1}{2}z + \infty\cdot n \;\rightarrow\; \frac{1}{2}z\,, \quad (2.2)$$

with all four points connected by straight lines. It is now important to realize that the integration contour of the Wilson line not only depends on the coordinates of the initial and final points but also on the light-cone direction which is opposite to the direction of P [33]. This induces a dependence on a light-cone vector n. In fact, instead of using n a rescaled vector λn with some positive parameter λ could be taken in order to specify the Wilson line. Therefore, the correlator actually only depends on the vector

$$N = \frac{M^2\, n}{P\cdot n}\,, \quad (2.3)$$

which is invariant under the mentioned rescaling. For convenience in (2.3) the hadron mass M is used such that N has the same mass dimension as an ordinary 4-momentum. The parameter η in (2.1) is defined through the zeroth component of n according to

$$\eta = \mathrm{sign}(n_0)\,, \quad (2.4)$$

which means that we simultaneously treat future-pointing ($\eta = +1$) and past-pointing ($\eta = -1$) Wilson lines. Keeping this dependence is particularly convenient once we make the projection of the correlator in (2.1) onto the correlator defining TMDs.

In order to obtain the parameterization of the correlator in (2.1) in terms of GPCFs we make use of the following constraints due to parity, hermiticity, and time reversal,

$$W(P,k,\Delta,N;\eta) = \gamma_0\, W(\bar{P},\bar{k},\bar{\Delta},\bar{N};\eta)\,\gamma_0\,, \quad (2.5)$$

$$W^\dagger(P,k,\Delta,N;\eta) = \gamma_0\, W(P,k,-\Delta,N;\eta)\,\gamma_0\,, \quad (2.6)$$

$$W^*(P,k,\Delta,N;\eta) = \big(-i\gamma_5 C\big)\, W(\bar{P},\bar{k},\bar{\Delta},\bar{N};-\eta)\,\big(-i\gamma_5 C\big)\,, \quad (2.7)$$

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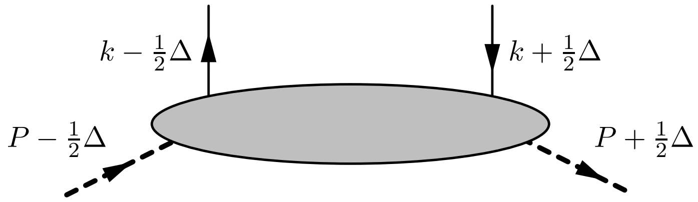

Figure 1: Kinematics for GPCFs.

where $\bar{P}^\mu = P_\mu = (P^0, -\vec{P})$ etc., while C is the charge conjugation matrix. It turns out that the general structure of the correlator W can already be obtained on the basis of the parity constraint in (2.5). One ends up with the following 16 linearly independent matrix structures multiplied by scalar functions,

$$\begin{aligned} W(P,k,\Delta,N;\eta) = {} & M A_1 + \not{P} A_2 + \not{k} A_3 + \not{\Delta} A_4 + \frac{[\not{P},\not{k}]}{2M} A_5 + \frac{[\not{P},\not{\Delta}]}{2M} A_6 + \frac{[\not{k},\not{\Delta}]}{2M} A_7 \\ & + \frac{i\varepsilon^{\mu\nu\rho\sigma}\gamma_\mu\gamma_5 P_\nu k_\rho \Delta_\sigma}{M^2} A_8 + \not{N} B_1 + \frac{[\not{P},\not{N}]}{2M} B_2 + \frac{[\not{k},\not{N}]}{2M} B_3 \\ & + \frac{[\not{\Delta},\not{N}]}{2M} B_4 + \frac{i\varepsilon^{\mu\nu\rho\sigma}\gamma_\mu\gamma_5 P_\nu k_\rho N_\sigma}{M^2} B_5 + \frac{i\varepsilon^{\mu\nu\rho\sigma}\gamma_\mu\gamma_5 P_\nu \Delta_\rho N_\sigma}{M^2} B_6 \\ & + \frac{i\varepsilon^{\mu\nu\rho\sigma}\gamma_\mu\gamma_5 k_\nu \Delta_\rho N_\sigma}{M^2} B_7 + \frac{i\varepsilon^{\mu\nu\rho\sigma}\gamma_5 P_\mu k_\nu \Delta_\rho N_\sigma}{M^3} B_8 \,. \end{aligned} \tag{2.8}$$

Our treatment leading to (2.8) is very similar to what has already been done for the quark-quark correlator in the case $\Delta = 0$ for a spin-0 hadron [33, 34] and a spin-1/2 hadron [35]. The functions A_i and B_i are independent and represent the GPCFs. The various factors of M are introduced in order to assign the same mass dimension to all GPCFs. We emphasize that one has to use the identity

$$g^{\alpha\beta}\varepsilon^{\mu\nu\rho\sigma} = g^{\mu\beta}\varepsilon^{\alpha\nu\rho\sigma} + g^{\nu\beta}\varepsilon^{\mu\alpha\rho\sigma} + g^{\rho\beta}\varepsilon^{\mu\nu\alpha\sigma} + g^{\sigma\beta}\varepsilon^{\mu\nu\rho\alpha} \,, \tag{2.9}$$

in order to eliminate redundant terms. While hermiticity and time-reversal do not affect the general structure of the correlator W, they impose constraints on the GPCFs. Applying the hermiticity constraint (2.6) to the decomposition in (2.8) one finds

$$X^*(P,k,\Delta,N;\eta) = \pm X(P,k,-\Delta,N;\eta) \,, \tag{2.10}$$

where the plus sign holds for $X = A_1$, A_2, A_3, A_6, A_7, A_8, B_1, B_4, B_6, B_7, and the minus sign for $X = A_4$, A_5, B_2, B_3, B_5, B_8. The time-reversal constraint (2.7) provides

$$X^*(P,k,\Delta,N;\eta) = X(P,k,\Delta,N;-\eta) \tag{2.11}$$

for all $X = A_i$, B_i, relating GPCFs defined with future-pointing Wilson lines to those defined with past-pointing lines. From these considerations it follows that GPCFs, unlike GPDs or TMDs, in general are complex-valued functions. Keeping now in mind that $\eta \in \{-1\,,1\}$ and using eq. (2.11) one finds immediately that only the imaginary part of the GPCFs depends on η. This allows one to write

$$X(P,k,\Delta,N;\eta) = X^e(P,k,\Delta,N) + i\,X^o(P,k,\Delta,N;\eta) \,, \tag{2.12}$$

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with

$$X^o(P,k,\Delta,N;\eta) = -X^o(P,k,\Delta,N;-\eta)\,, \tag{2.13}$$

where we call X^e the T-even and X^o the T-odd part of the generic GPCF X. The sign reversal of X^o in eq. (2.13) when going from future-pointing to past-pointing Wilson lines corresponds to the sign reversal discussed in ref. [36] for T-odd TMDs.

Now we would like to give a first account on the relation between GPCFs on the one hand and GPDs as well as TMDs on the other. To this end we consider the quark-quark correlator F defining GPDs for a spin-0 target, which can be obtained from the correlator W in eq. (2.1) by means of the projection

$$\begin{aligned} F_{ij}(P,x,\Delta,N) &= \int dk^-\, d^2\vec{k}_T\, W_{ij}(P,k,\Delta,N;\eta) \\ &= \int \frac{dz^-}{2\pi}\, e^{ik\cdot z}\, \langle p' \,|\, \bar{\psi}_j\Big(-\frac{1}{2}z\Big)\, \mathcal{W}\Big(-\frac{1}{2}z, \frac{1}{2}z \,\Big|\, n\Big)\, \psi_i\Big(\frac{1}{2}z\Big)\, |\, p\rangle \Big|_{z^+=\vec{z}_T=0} \quad . \end{aligned} \tag{2.14}$$

In this formula use is made of light-cone components that are specified according to $a^\pm = (a^0 \pm a^3)/\sqrt{2}$ and $\vec{a}_T = (a^1, a^2)$ for a generic 4-vector $a = (a^0, a^1, a^2, a^3)$, where, in particular, we choose $k^+ = xP^+$. Note that after integrating upon k^- and $\vec{k}_T$ the dependence on the parameter η drops out. As is well-known, in this case we are dealing with a light-cone correlator and the two quark fields are just connected by a straight line. This means that the choice of the contour in (2.2) leads, after projection, to the appropriate Wilson line for the GPD-correlator.

The correlator Φ defining TMDs can be extracted from W by putting $\Delta = 0$ and integrating out one light-cone component of the quark momentum (which we choose to be k^-),

$$\begin{aligned} \Phi_{ij}(P,x,\vec{k}_T,N;\eta) &= \int dk^-\, W_{ij}(P,k,0,N;\eta) \\ &= \int \frac{dz^-\, d^2\vec{z}_T}{(2\pi)^3}\, e^{ik\cdot z}\, \langle P \,|\, \bar{\psi}_j\Big(-\frac{1}{2}z\Big)\, \mathcal{W}\Big(-\frac{1}{2}z, \frac{1}{2}z \,\Big|\, n\Big)\, \psi_i\Big(\frac{1}{2}z\Big)\, |\, P\rangle \Big|_{z^+=0} \quad . \end{aligned} \tag{2.15}$$

Note that for $\Delta = 0$ one has $p = p' = P$. We point out that the path specified in (2.2) also leads to a proper Wilson line after taking the TMD-limit [37–39, 36, 40–44]. Since Φ in eq. (2.15) is not a light-cone correlator the dependence on the parameter η remains. The case $\eta = +1$ is appropriate for defining TMDs in processes with final state interactions of the struck quark like SIDIS, while $\eta = -1$ can be used for TMDs in DY [36]. It has been emphasized in refs. [37, 45, 46] that, in general, light-like Wilson lines as used in the unintegrated correlators in (2.1) and (2.15) lead to divergences. Such divergences can be avoided, however, by adopting a near light-cone direction. For the purpose of the present work it is sufficient to note that our general reasoning remains valid once a near light-cone direction is used instead of n.

It is evident that not only the correlators F and Φ appear as projections of the most general two-parton correlator W as outlined above, but also the GPDs and the TMDs are projections of certain GPCFs. Therefore, GPCFs can be considered as *mother distributions*, which actually contain the maximum amount of information on the two-parton structure of hadrons [24, 25, 4]. Despite this fact a classification of the GPCFs as given in (2.8) has never been worked out.

3. Generalized transverse momentum dependent parton distributions

The projections in (2.14) and (2.15) contain the integration upon the minus-component of the quark momentum. Therefore, it is useful to consider in more detail the correlator

$$\begin{aligned} W_{ij}(P,x,\vec{k}_T,\Delta,N;\eta) &= \int dk^-\, W_{ij}(P,k,\Delta,N;\eta) \\ &= \int \frac{dz^-\, d^2\vec{z}_T}{(2\pi)^3}\, e^{ik\cdot z}\, \langle p'\,|\,\bar{\psi}_j\Big(-\frac{1}{2}z\Big)\,\mathcal{W}\Big(-\frac{1}{2}z,\frac{1}{2}z\,\Big|\,n\Big)\,\psi_i\Big(\frac{1}{2}z\Big)\,|\,p\rangle\Big|_{z^+=0} \; . \end{aligned} \tag{3.1}$$

Below the parameterization of this object is given in terms of what we call generalized transverse momentum dependent parton distributions (GTMDs). Of course this result can now be obtained in a straightforward manner on the basis of the decomposition in eq. (2.8). From the discussion we have provided so far it is obvious that also the GTMDs, like the GPCFs, can be considered as *mother distributions* of GPDs and TMDs. It is the correlator in (3.1) which for instance can enter the description of hard exclusive meson production [29], while the corresponding correlator for gluons appears when considering diffractive processes in lepton-hadron as well as hadron-hadron collisions [31, 32]. The question whether or not it appears with a Wilson line as defined in (2.2) to our knowledge has never been addressed in the literature and requires further investigation that goes beyond the scope of the present work.

The matrix of the generalized k_T-dependent correlator in (3.1) is fully specified by means of all possible independent Dirac traces which we denote by

$$\begin{aligned} W^{[\Gamma]}(P,x,\vec{k}_T,\Delta,N;\eta) &= \frac{1}{2}\,\mathrm{Tr}\big[W(P,x,\vec{k}_T,\Delta,N;\eta)\,\Gamma\big] \\ &= \int \frac{dz^-\, d^2\vec{z}_T}{2(2\pi)^3}\, e^{ik\cdot z}\, \langle p'\,|\,\bar{\psi}\Big(-\frac{1}{2}z\Big)\,\Gamma\,\mathcal{W}\Big(-\frac{1}{2}z,\frac{1}{2}z\,\Big|\,n\Big)\,\psi\Big(\frac{1}{2}z\Big)\,|\,p\rangle\Big|_{z^+=0} \; . \end{aligned} \tag{3.2}$$

In particular, in order to obtain a twist-classification for the GTMDs it is convenient to make use of the traces in eq. (3.2). We choose an infinite momentum frame such that P has a large plus-momentum and no transverse momentum. The plus-component of Δ is expressed through the commonly used variable ξ. To be now precise the 4-momenta in (2.8) are specified according to

$$P = \Big[\, P^+ \,,\, \frac{4M^2+\vec{\Delta}_T^2}{8(1-\xi^2)P^+} \,,\, \vec{0}_T \Big]\,, \tag{3.3}$$

$$k = \Big[\, xP^+ \,,\, k^- \,,\, \vec{k}_T \Big]\,, \tag{3.4}$$

$$\Delta = \Big[\, -2\xi P^+ \,,\, \frac{4\xi M^2+\xi\vec{\Delta}_T^2}{4(1-\xi^2)P^+} \,,\, \vec{\Delta}_T \Big]\,, \tag{3.5}$$

$$n = \Big[\, 0 \,,\, \pm 1 \,,\, \vec{0}_T \Big]\,. \tag{3.6}$$

The vector n in eq. (3.6) is of course not the most general light-cone vector. In particular, it has no transverse component and points opposite to the direction of P as already mentioned earlier. However, if one wants to arrive at an appropriate definition of TMDs for SIDIS and DY, there is no freedom left for this vector because it is fixed by the external momenta of the processes.

Now we have all the ingredients necessary to write down the final result for the traces of the generalized k_T-dependent correlator (3.1) in terms of GTMDs. We start with the twist-2 case for which one gets

$$W^{[\gamma^+]} = F_1\,, \tag{3.7}$$

$$W^{[\gamma^+\gamma_5]} = \frac{i\varepsilon_T^{ij} k_T^i \Delta_T^j}{M^2}\,\tilde{G}_1\,, \tag{3.8}$$

$$W^{[i\sigma^{j+}\gamma_5]} = \frac{i\varepsilon_T^{ij} k_T^i}{M}\,H_1^k + \frac{i\varepsilon_T^{ij} \Delta_T^i}{M}\,H_1^\Delta\,. \tag{3.9}$$

Here the definitions $\varepsilon^{0123} = 1$ and $\varepsilon_T^{ij} = \varepsilon^{-+ij}$ as well as the standard notation $\sigma^{\mu\nu} = i[\gamma^\mu, \gamma^\nu]/2$ are used. The four complex-valued twist-2 GTMDs F_1, $\tilde{G}_1$, H_1^k, H_1^Δ are given by k^--integrals of certain linear combinations of the GPCFs in (2.8), where the explicit relations are listed in appendix A for all twists. To shorten the notation the arguments on both sides of the eqs. (3.7)–(3.9) are omitted. All GTMDs depend on the set of variables $(x, \xi, \vec{k}_T^2, \vec{k}_T \cdot \vec{\Delta}_T, \vec{\Delta}_T^2; \eta)$.

In the twist-3 case, characterized through a suppression by one power in P^+, we find

$$W^{[1]} = \frac{M}{P^+}\left[E_2\right], \tag{3.10}$$

$$W^{[\gamma_5]} = \frac{M}{P^+}\left[\frac{i\varepsilon_T^{ij} k_T^i \Delta_T^j}{M^2}\,\tilde{E}_2\right], \tag{3.11}$$

$$W^{[\gamma^j]} = \frac{M}{P^+}\left[\frac{k_T^j}{M}\,F_2^k + \frac{\Delta_T^j}{M}\,F_2^\Delta\right], \tag{3.12}$$

$$W^{[\gamma^j\gamma_5]} = \frac{M}{P^+}\left[\frac{i\varepsilon_T^{ij} k_T^i}{M}\,G_2^k + \frac{i\varepsilon_T^{ij} \Delta_T^i}{M}\,G_2^\Delta\right], \tag{3.13}$$

$$W^{[i\sigma^{ij}\gamma_5]} = \frac{M}{P^+}\left[i\varepsilon_T^{ij} H_2\right], \tag{3.14}$$

$$W^{[i\sigma^{+-}\gamma_5]} = \frac{M}{P^+}\left[\frac{i\varepsilon_T^{ij} k_T^i \Delta_T^j}{M^2}\,\tilde{H}_2\right]. \tag{3.15}$$

The twist-4 result, which is basically a copy of the twist-2 case, reads

$$W^{[\gamma^-]} = \frac{M^2}{(P^+)^2}\left[F_3\right], \tag{3.16}$$

$$W^{[\gamma^-\gamma_5]} = \frac{M^2}{(P^+)^2}\left[\frac{i\varepsilon_T^{ij} k_T^i \Delta_T^j}{M^2}\,\tilde{G}_3\right], \tag{3.17}$$

$$W^{[i\sigma^{j-}\gamma_5]} = \frac{M^2}{(P^+)^2}\left[\frac{i\varepsilon_T^{ij} k_T^i}{M}\,H_3^k + \frac{i\varepsilon_T^{ij} \Delta_T^i}{M}\,H_3^\Delta\right]. \tag{3.18}$$

The twist-4 case is of course at most of academic interest but is included for completeness.

Like in the case of the GPCFs we also consider the implications of hermiticity and time-reversal on the GTMDs. Hermiticity leads to

$$X^*(x,\xi,\vec{k}_T^2,\vec{k}_T\cdot\vec{\Delta}_T,\vec{\Delta}_T^2;\eta) = \pm X(x,-\xi,\vec{k}_T^2,-\vec{k}_T\cdot\vec{\Delta}_T,\vec{\Delta}_T^2;\eta)\,, \tag{3.19}$$

with a plus sign for $X = E_2,\ F_1,\ F_2^k,\ F_3,\ \tilde{G}_1,\ G_2^\Delta,\ \tilde{G}_3,\ H_1^\Delta,\ \tilde{H}_2,\ H_3^\Delta$, and the minus sign for $X = \tilde{E}_2,\ F_2^\Delta,\ G_2^k,\ H_1^k,\ H_2,\ H_3^k$. These results are a direct consequence of (2.10) and the relations in eqs. (A.1)–(A.16). On the basis of (2.11) one obtains from time-reversal

$$X^*(x,\xi,\vec{k}_T^2,\vec{k}_T\cdot\vec{\Delta}_T,\vec{\Delta}_T^2;\eta) = X(x,\xi,\vec{k}_T^2,\vec{k}_T\cdot\vec{\Delta}_T,\vec{\Delta}_T^2;-\eta)\,. \tag{3.20}$$

for all GTMDs X. This means, in particular, that we can carry over eqs. (2.12) and (2.13) to the GTMD case and write

$$X(x,\xi,\vec{k}_T^2,\vec{k}_T\cdot\vec{\Delta}_T,\vec{\Delta}_T^2;\eta) = X^e(x,\xi,\vec{k}_T^2,\vec{k}_T\cdot\vec{\Delta}_T,\vec{\Delta}_T^2)+i\,X^o(x,\xi,\vec{k}_T^2,\vec{k}_T\cdot\vec{\Delta}_T,\vec{\Delta}_T^2;\eta)\,, \tag{3.21}$$

with the real valued functions X^e and X^o respectively representing the real and imaginary part of the GTMD X. Only the T-odd part X^o depends on the sign of η according to

$$X^o(x,\xi,\vec{k}_T^2,\vec{k}_T\cdot\vec{\Delta}_T,\vec{\Delta}_T^2;\eta) = -X^o(x,\xi,\vec{k}_T^2,\vec{k}_T\cdot\vec{\Delta}_T,\vec{\Delta}_T^2;-\eta)\,, \tag{3.22}$$

i.e., the imaginary parts of GTMDs defined with future-pointing and past-pointing Wilson lines have a reversed sign.

In order to give an estimate we have calculated the GTMDs in a simple spectator model of the pion. The results are presented in appendix B. Our treatment is restricted to lowest order in perturbation theory. To this order all T-odd parts of the GTMDs vanish — a feature which is also well-known from spectator model calculations of T-odd TMDs. All the results listed in eqs. (B.5)–(B.20) are in accordance with the hermiticity constraint (3.19).

4. Projection of GTMDs onto TMDs and GPDs

In this section we consider the generalized k_T-dependent correlator in eq. (3.1) for the specific TMD-kinematics and the GPD-kinematics. This procedure provides the relations between the *mother distributions* (GTMDs) on the one hand and the TMDs as well as GPDs on the other. On the basis of these results one can check whether there exists model-independent support for possible nontrivial relations between GPDs and TMDs.

4.1 TMD-limit

We start with the TMD-limit corresponding to a vanishing momentum transfer $\Delta = 0$. In this limit exactly half of the real-valued distributions vanish because they are odd as function of Δ due to the hermiticity constraint (3.19): $E_2^o,\ \tilde{E}_2^e,\ F_1^o,\ F_2^{k,o},\ F_2^{\Delta,e},\ F_3^o,\ \tilde{G}_1^o,\ G_2^{k,e},\ G_2^{\Delta,o},\ \tilde{G}_3^o,\ H_1^{k,e},\ H_1^{\Delta,o},\ H_2^e,\ \tilde{H}_2^o,\ H_3^{k,e},\ H_3^{\Delta,o}$. In addition, the distributions $\tilde{E}_2^o,\ F_2^{\Delta,o},\ \tilde{G}_1^e,\ G_2^{\Delta,e},\ \tilde{G}_3^e,\ H_1^{\Delta,e},\ \tilde{H}_2^e,\ H_3^{\Delta,e}$ do not appear in the correlator any more, because they are multiplied by a coefficient which is linear in Δ. Therefore, in the TMD-limit only the following eight (four T-even and four T-odd) distributions survive: E_2^e, F_1^e, $F_2^{k,e}$, F_3^e, $G_2^{k,o}$, $H_1^{k,o}$, H_2^o, $H_3^{k,o}$. The complete list of TMDs for a spin-1/2 hadron has been given in ref. [35][1] (see also the review article [11]). Considering in [35] the limit of a spinless target one ends up with eight TMDs which agrees with the number of GTMDs in the limit $\Delta = 0$. Using for the TMDs the notation of [35] one finds the following explicit relations between the TMDs and the GTMDs:

$$f_1(x, \vec{k}_T^2) = F_1^e(x, 0, \vec{k}_T^2, 0, 0)\,, \tag{4.1}$$
$$h_1^\perp(x, \vec{k}_T^2; \eta) = H_1^{k,o}(x, 0, \vec{k}_T^2, 0, 0; \eta)\,, \tag{4.2}$$
$$e(x, \vec{k}_T^2) = E_2^e(x, 0, \vec{k}_T^2, 0, 0)\,, \tag{4.3}$$
$$f^\perp(x, \vec{k}_T^2) = F_2^{k,e}(x, 0, \vec{k}_T^2, 0, 0)\,, \tag{4.4}$$
$$g^\perp(x, \vec{k}_T^2; \eta) = G_2^{k,o}(x, 0, \vec{k}_T^2, 0, 0; \eta)\,, \tag{4.5}$$
$$h(x, \vec{k}_T^2; \eta) = H_2^o(x, 0, \vec{k}_T^2, 0, 0; \eta)\,, \tag{4.6}$$
$$f_3(x, \vec{k}_T^2) = F_3^e(x, 0, \vec{k}_T^2, 0, 0)\,, \tag{4.7}$$
$$h_3^\perp(x, \vec{k}_T^2; \eta) = H_3^{k,o}(x, 0, \vec{k}_T^2, 0, 0; \eta)\,. \tag{4.8}$$

These results are obtained by means of eqs. (2.15), (3.1), and (3.2), together with the explicit expressions for the traces of the TMD-correlator Φ in terms of TMDs as given in [35], and the traces of the GTMD-correlator in (3.7)–(3.18). The four TMDs $h_1^\perp$, $g^\perp$, h, $h_3^\perp$ are T-odd and are related to T-odd GTMDs.

4.2 GPD-limit

In a second step we concentrate on the GPD-limit which appears when integrating upon the transverse parton momentum $\vec{k}_T$. As already discussed after eq. (2.14) the dependence on η drops out in this case which implies, in particular, that all effects of T-odd GTMDs disappear.

In the literature only the twist-2 and the chiral-even twist-3 GPDs for a spin-0 target have been introduced [47, 4]. Therefore, we give here for the first time a complete list of GPDs for all twists. The GPDs parameterize the Dirac traces $F^{[\Gamma]}$ of the GPD-correlator in (2.14). One finds two nonvanishing traces for twist-2, four traces for twist-3, and two traces for twist-4. To be explicit the GPDs can be defined according to

$$F^{[\gamma^+]} = F_1^\pi(x, \xi, t)\,, \tag{4.9}$$
$$F^{[i\sigma^{j+}\gamma_5]} = \frac{i\varepsilon_T^{ij}\Delta_T^i}{M}\, H_1^\pi(x, \xi, t)\,, \tag{4.10}$$
$$F^{[1]} = \frac{M}{P^+}\Big[E_2^\pi(x, \xi, t)\Big]\,, \tag{4.11}$$
$$F^{[\gamma^j]} = \frac{M}{P^+}\Big[\frac{\Delta_T^j}{M}\, F_2^\pi(x, \xi, t)\Big]\,, \tag{4.12}$$

[1] Note that the *l.h.s.* in eqs. (16) and (25) of [35] should read $\Phi^{[i\sigma^{i+}\gamma_5]}$ and $\Phi^{[i\sigma^{i-}\gamma_5]}$, respectively.

$$F^{[\gamma^j\gamma_5]} = \frac{M}{P^+}\left[\frac{i\varepsilon_T^{ij}\Delta_T^i}{M}\, G_2^\pi(x,\xi,t)\right], \tag{4.13}$$

$$F^{[i\sigma^{ij}\gamma_5]} = \frac{M}{P^+}\left[i\varepsilon_T^{ij} H_2^\pi(x,\xi,t)\right], \tag{4.14}$$

$$F^{[\gamma^-]} = \frac{M^2}{(P^+)^2}\left[F_3^\pi(x,\xi,t)\right], \tag{4.15}$$

$$F^{[i\sigma^{j-}\gamma_5]} = \frac{M^2}{(P^+)^2}\left[\frac{i\varepsilon_T^{ij}\Delta_T^i}{M}\, H_3^\pi(x,\xi,t)\right], \tag{4.16}$$

where $t = \Delta^2$. The structure of the traces in (4.9)–(4.16) follows readily from eqs. (3.7)–(3.18) if one keeps in mind that after integrating upon $\vec{k}_T$ the only transverse vector left is $\vec{\Delta}_T$. Altogether there exist eight GPDs corresponding to the number of TMDs for a spin-0 hadron. The four GPDs F_1^π, F_2^π, G_2^π, F_3^π are chiral-even, while the remaining ones are chiral-odd. In the next subsection we consider in more detail the GPD H_1^π which is related to the object $\bar{E}_T$ used in [48] according to $H_1^\pi = -\bar{E}_T$. The twist-3 GPDs F_2^π and G_2^π are related to the functions H_3 and H_A that were introduced in ref. [47]. At this point it is also worthwhile to notice that, in general, measuring chiral-odd GPDs would be a demanding task. For instance in the case of hard exclusive production of a single meson one would have to consider subleading twist observables in order to get access to these objects [49, 50]. Alternatively one could resort to relatively complicated processes like diffractive electroproduction of two mesons [51]. On the other hand chiral-odd GPDs can well be investigated using lattice QCD [52] or models of the nonperturbative strong interaction.

It is now straightforward to write down the following expressions for the GPDs in terms of k_T-integrals of GTMDs:

$$F_1^\pi(x,\xi,t) = \int d^2\vec{k}_T\, F_1^e\,, \tag{4.17}$$

$$H_1^\pi(x,\xi,t) = \int d^2\vec{k}_T\left[\frac{\vec{k}_T\cdot\vec{\Delta}_T}{\vec{\Delta}_T^2}\, H_1^{k,e} + H_1^{\Delta,e}\right], \tag{4.18}$$

$$E_2^\pi(x,\xi,t) = \int d^2\vec{k}_T\, E_2^e\,, \tag{4.19}$$

$$F_2^\pi(x,\xi,t) = \int d^2\vec{k}_T\left[\frac{\vec{k}_T\cdot\vec{\Delta}_T}{\vec{\Delta}_T^2}\, F_2^{k,e} + F_2^{\Delta,e}\right], \tag{4.20}$$

$$G_2^\pi(x,\xi,t) = \int d^2\vec{k}_T\left[\frac{\vec{k}_T\cdot\vec{\Delta}_T}{\vec{\Delta}_T^2}\, G_2^{k,e} + G_2^{\Delta,e}\right], \tag{4.21}$$

$$H_2^\pi(x,\xi,t) = \int d^2\vec{k}_T\, H_2^e\,, \tag{4.22}$$

$$F_3^\pi(x,\xi,t) = \int d^2\vec{k}_T\, F_3^e\,, \tag{4.23}$$

$$H_3^\pi(x,\xi,t) = \int d^2\vec{k}_T\left[\frac{\vec{k}_T\cdot\vec{\Delta}_T}{\vec{\Delta}_T^2}\, H_3^{k,e} + H_3^{\Delta,e}\right]. \tag{4.24}$$

Note that in eqs. (4.17)–(4.24) the limit $\vec{\Delta}_T \to 0$ can be performed without encountering a singularity because of

$$\int d^2\vec{k}_T\, k_T^i\, X(x,\xi,\vec{k}_T^2,\vec{k}_T\cdot\vec{\Delta}_T,\vec{\Delta}_T^2;\eta) \propto \Delta_T^i\,, \tag{4.25}$$

which holds for any GTMD X. The hermiticity constraint (3.19) for the GTMDs, in combination with the relations (4.17)–(4.24), determines the symmetry behavior of the GPDs under the transformation $\xi \to -\xi$. One finds that the six GPDs E_2^π, F_1^π, F_3^π, G_2^π, H_1^π, H_3^π are even functions in ξ, while F_2^π and H_2^π are odd in ξ. This implies

$$F_2^\pi(x,0,t) = 0\,, \qquad H_2^\pi(x,0,t) = 0\,. \tag{4.26}$$

In the following subsection we will make use of (4.26).

4.3 Relations between GPDs and TMDs

Having established the precise connection of the GPDs and TMDs with their respective *mother distributions* we are now in a position to search for possible model-independent relations between GPDs and TMDs. From (4.17) and (4.1) it is obvious that the GPD F_1^π and the TMD f_1 can be related since both functions are projections of the GTMD F_1^e. With an analogous reasoning also one relation between twist-3 and one between twist-4 distributions can be obtained leading altogether to

$$F_1^\pi(x,0,0) = \int d^2\vec{k}_T\, F_1^e(x,0,\vec{k}_T^2,0,0) = \int d^2\vec{k}_T\, f_1(x,\vec{k}_T^2) = f_1(x)\,, \tag{4.27}$$

$$E_2^\pi(x,0,0) = \int d^2\vec{k}_T\, E_2^e(x,0,\vec{k}_T^2,0,0) = \int d^2\vec{k}_T\, e(x,\vec{k}_T^2) = e(x)\,, \tag{4.28}$$

$$F_3^\pi(x,0,0) = \int d^2\vec{k}_T\, F_3^e(x,0,\vec{k}_T^2,0,0) = \int d^2\vec{k}_T\, f_3(x,\vec{k}_T^2) = f_3(x)\,. \tag{4.29}$$

These formulas have the same status as the (trivial) model-independent relations between GPDs and TMDs which are known for certain twist-2 quark and gluon distributions of the nucleon (called relations of first type in ref. [20]).

However, also nontrivial relations between GPDs and TMDs have been suggested in the literature [14−20] in the case of a spin-1/2 target. So far these relations have only been established in low-order calculations in the framework of simple spectator models. Our GTMD-analysis can now shed light on the question if model-independent nontrivial relations exist.

In the case of a nucleon target it was shown that for instance the Boer-Mulders function $h_1^\perp$ has a (model-dependent) nontrivial relation to a certain linear combination of chiral-odd GPDs [17, 18, 20]. This result suggests for a spin-0 target a relation between $h_1^\perp$ and H_1^π. We can investigate this issue using the simple spectator model of the pion discussed in appendix B. The GPD H_1^π is obtained by means of eq. (4.18) and the results for the GTMDs in (B.15) and (B.16). One finds

$$H_1^\pi\left(x,0,-\frac{\vec{\Delta}_T^2}{(1-x)^2}\right) = -\frac{g_\pi^2\,(1-x)}{(2\pi)^3}\int d^2\vec{k}_T\,\frac{m\,M}{[\vec{k}_T^2+\tilde{M}^2(x)][(\vec{k}_T+\vec{\Delta}_T)^2+\tilde{M}^2(x)]}\,,$$
$$\text{with}\qquad \tilde{M}^2(x) = m^2 - x(1-x)M^2\,. \tag{4.30}$$

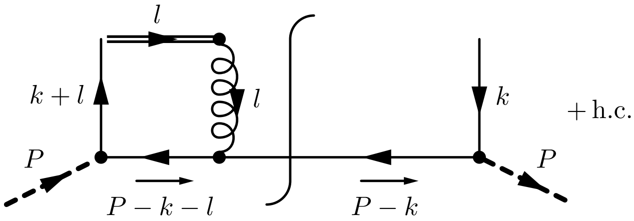

Figure 2: Lowest nontrivial order diagram for T-odd TMDs in the spectator model for a pion. The Hermitian conjugate diagram (h.c.) is not shown. The eikonal propagator arising from the Wilson line in the operator definition of TMDs is indicated by a double line.

Note that, in general, relations between GPDs and TMDs always invoke GPDs that are evaluated at the specific kinematical point $\xi = 0$ [20]. A nonzero Boer-Mulders function may be generated by including contributions from the Wilson line in the TMD correlator. To lowest nontrivial order one has to consider the diagram shown in figure 2, where a gluon is exchanged between the high-energy (eikonalized) quark and the spectator parton. (For related treatments of the Boer-Mulders function of the nucleon we refer to [53−57].) In the case of valence quarks and antiquarks of a neutral pion we find[2]

$$h_1^\perp(x, \vec{k}_T^2; \eta) = -\eta \, \frac{2 g_\pi^2 \, g_s^2}{3(2\pi)^4} \, \frac{m \, M}{\vec{k}_T^2 \, (\vec{k}_T^2 + \tilde{M}^2(x))} \, \ln\left(\frac{\vec{k}_T^2 + \tilde{M}^2(x)}{\tilde{M}^2(x)} \right), \tag{4.31}$$

with g_s denoting the strong coupling. One has to multiply the expression in (4.31) by two in order to get the valence Boer-Mulders functions of charged pions. Obviously $h_1^\perp$ is negative which agrees with previous expectations [48]. Using straightforward manipulations one can now show that the distributions in (4.30) and (4.31) are related according to

$$\frac{2 g_s^2}{3(2\pi)^2 \, (1-x)} \int d^2\vec{\Delta}_T \, H_1^\pi\left(x, 0, -\frac{\vec{\Delta}_T^2}{(1-x)^2} \right) = \eta \int d^2\vec{k}_T \, \frac{\vec{k}_T^2}{2M^2} \, h_1^\perp(x, \vec{k}_T^2; \eta) \,. \tag{4.32}$$

This equation is of the form of the nontrivial relation of second type presented in eq. (106) of ref. [20]. We point out that there exist different ways of writing the relation between H_1^π and h_1. For instance one may transform the GPD into impact parameter space which leads to an interesting physical interpretation of the relation [14, 16, 17, 20]. Moreover, relations invoking different Δ_T-moments of GPDs and k_T-moments of TMDs can also be established [19, 20]. For the purpose of our discussion here it is sufficient to notice that in the framework of a simple spectator model of the pion a nontrivial relation between H_1^π and $h_1^\perp$ holds.

On the other hand, the GTMD-analysis does not support a model-independent status of such a relation because, according to (4.18) and (4.2), H_1^π and $h_1^\perp$ have different, independent *mother distributions*. One may wonder if the specific kinematical point $\xi = 0$ and the use of moments as taken in (4.32) spoils the argument. However, it turns out that

[2] A calculation of $h_1^\perp$ of the pion using the pseudo-scalar model defined in appendix B has already been performed in ref. [58], but the result quoted in that paper does not fully agree with ours. Both results differ, in particular, by a factor $\frac{1}{2}(1-x)$.

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the contributions of all the involved GTMDs survive these operations. Unless, for some reason, the involved GTMDs are subject to further constraints one has to conclude that there cannot exist a model-independent relation between H_1^π and $h_1^\perp$. This conclusion is in accordance with the observation made in [20] that nontrivial relations of second type are likely to even break down in spectator models once higher order contributions are taken into account. Therefore, one has to attribute the relation to the simplicity of the used model. Nevertheless, it may well be that numerically the model-dependent nontrivial relation works reasonably well when comparing to experimental data. In fact such a case is already known for distributions of the nucleon, namely the relation between the Sivers function $f_{1T}^\perp$ and the GPD E [14 − 16, 20].

In order to continue we now note that according to eq. (4.26) the GPDs F_2^π and H_2^π vanish for $\xi = 0$. Therefore, they cannot be related to TMDs. This means we are just left with the GPDs G_2^π and H_3^π. Since their potential counterparts $g^\perp$ and $h_3^\perp$ on the TMD side are again related to different GTMDs also in that case no model-independent relation exists. Eventually, we mention that for the subleading twist functions we have not explored whether any model-dependent nontrivial relation can be obtained.

5. Conclusions

In summary, we have derived the structure of the fully unintegrated, off-diagonal quark-quark correlator for a spin-0 hadron. This object, which contains the most general information on the two-parton structure of a hadron, has been parameterized in terms of so-called generalized parton correlation functions (GPCFs). Integrating the GPCFs upon a light-cone component of the quark momentum one ends up with entities which we called generalized transverse momentum dependent parton distributions (GTMDs). In general, GTMDs can be of direct relevance for the phenomenology of various hard (diffractive) processes (see, e.g., refs. [31, 32, 29]). Our analysis shows that both the GPCFs and the GTMDs in general are complex-valued functions. This is different from the (simpler) forward parton distributions, GPDs, and TMDs all of which are real.

Suitable projections of GTMDs lead to GPDs on the one hand and TMDs on the other. Therefore, GTMDs can be considered as *mother distributions* of GPDs and TMDs [24, 25, 4]. To study these two limiting cases of GTMDs was the main motivation of the present work. One outcome was the first complete classification of GPDs beyond leading twist. Most importantly, we were able to determine which of the GPDs and TMDs have the same *mother distributions* allowing us to explore whether model-independent relations between GPDs and TMDs can be established. For a spin-0 hadron one ends up with three such model-independent relations. Actually, these cases can be considered as trivial ones because the respective GPDs and TMDs also have a relation to the same forward parton distributions (see also ref. [20]). Our main interest was to investigate nontrivial relations between GPDs and TMDs which have been obtained in simple spectator models and extensively discussed in the recent literature [14 − 20]. We have elaborated in some detail on a possible relation between the (twist-2) Boer-Mulders function $h_1^\perp$ of a pion and a chiral-odd GPD (denoted by H_1^π). While a (nontrivial) relation can be found in a low order calculation using a simple spectator model of the pion, the GTMD-analysis shows that such a relation cannot have a model-independent status because $h_1^\perp$ and H_1^π are related to different, independent *mother distributions*. This finding agrees with ref. [20] where it has been argued that nontrivial relations between GPDs and TMDs are likely to break down even in spectator models if the parton distributions are evaluated to higher order in perturbation theory. Altogether we found that only the three mentioned trivial relations can be model-independent. We emphasize that this conclusion does not tell anything about the numerical violation of possible nontrivial relations between GPDs and TMDs.

The present work should be extended in various directions. First of all, it would be worthwhile to repeat our analysis for the more interesting but at the same time also more complicated case of a nucleon target. Moreover, one should focus on the phenomenology of GPCFs and GTMDs. For instance, in this context it has to be clarified if the GTMDs in physical processes appear with the Wilson line as defined in our work. To our knowledge previous articles using GTMDs did not address this important question. In addition, if one wants to study diffractive reactions, gluon GTMDs rather than quark GTMDs are relevant in a first place. We hope to return to these topics in future work.

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A. Relations between GTMDs and GPCFs

Here the explicit relations between the GTMDs in eqs. (3.7)–(3.18) and the GPCFs in eq. (2.8) are listed. For brevity we leave out the arguments of the functions. Straightforward calculation leads to the results

$$E_2 = 2P^+ \int dk^- \Big[A_1\Big], \tag{A.1}$$

$$\tilde{E}_2 = 2P^+ \int dk^- \Big[B_8\Big], \tag{A.2}$$

$$F_1 = 2P^+ \int dk^- \Big[A_2 + xA_3 - 2\xi A_4\Big], \tag{A.3}$$

$$F_2^k = 2P^+ \int dk^- \Big[A_3\Big], \tag{A.4}$$

$$F_2^\Delta = 2P^+ \int dk^- \Big[A_4\Big], \tag{A.5}$$

$$F_3 = 2P^+ \int dk^- \left[\frac{P^2}{2M^2}\left(A_2 - xA_3 + 2\xi A_4\right) + \frac{P\cdot k}{M^2}\, A_3 + B_1\right], \tag{A.6}$$

$$\tilde{G}_1 = 2P^+ \int dk^- \Big[A_8 \Big] \,, \tag{A.7}$$

$$G_2^k = 2P^+ \int dk^- \bigg[\frac{2\xi P^2}{M^2} A_8 + B_5 + 2\xi B_7 \bigg] \,, \tag{A.8}$$

$$G_2^\Delta = 2P^+ \int dk^- \bigg[\frac{xP^2 - P\cdot k}{M^2} A_8 + B_6 + xB_7 \bigg] \,, \tag{A.9}$$

$$\tilde{G}_3 = 2P^+ \int dk^- \bigg[-\frac{P^2}{2M^2} A_8 - B_7 \bigg] \,, \tag{A.10}$$

$$H_1^k = 2P^+ \int dk^- \Big[-A_5 - 2\xi A_7 \Big] \,, \tag{A.11}$$

$$H_1^\Delta = 2P^+ \int dk^- \Big[-A_6 - xA_7 \Big] \,, \tag{A.12}$$

$$\begin{aligned} H_2 = 2P^+ \int dk^- \bigg[& \frac{P^2}{M^2} \big(-xA_5 + 2\xi A_6 \big) + \frac{P\cdot k}{M^2} \big(A_5 + 2\xi A_7 \big) \\ & + B_2 + xB_3 - 2\xi B_4 \bigg] \,, \end{aligned} \tag{A.13}$$

$$\tilde{H}_2 = 2P^+ \int dk^- \Big[-A_7 \Big] \,, \tag{A.14}$$

$$H_3^k = 2P^+ \int dk^- \bigg[\frac{P^2}{2M^2} \big(A_5 - 2\xi A_7 \big) - B_3 \bigg] \,, \tag{A.15}$$

$$H_3^\Delta = 2P^+ \int dk^- \bigg[\frac{P^2}{2M^2} \big(A_6 - xA_7 \big) + \frac{P\cdot k}{M^2} A_7 - B_4 \bigg] \,. \tag{A.16}$$

B. Model calculation of GTMDs

For illustrative purposes and in order to get a first estimate we calculate all GTMDs for a spin-0 hadron in a simple spectator model by restricting ourselves to lowest nontrivial order in perturbation theory. To this order only two types of particles have to be considered: the meson (pion) target with mass M and quarks/antiquarks with mass m. In this model a pion, characterized by the field φ, is coupled to a quark and an antiquark by means of a pseudo-scalar interaction. Including isospin the interaction part of the Lagrangian reads

$$\mathcal{L}_{\rm int}(x) = -i\, g_\pi \, \bar{\Psi}(x)\, \gamma_5 \, \vec{\tau} \cdot \vec{\varphi}(x)\, \Psi(x) \,, \tag{B.1}$$

with the coupling constant g_π and the Pauli matrices τ_i. The results given below, which are valid for the kinematical range $0 \leq x \leq 1$, represent the valence quark and antiquark GTMDs for neutral pions. In order to get the valence GTMDs for charged pions one has to multiply the expressions by a factor two.

The lowest order contribution to the generalized k_T-dependent correlator in eq. (3.1) comes from the tree-level diagram depicted in figure 3. This diagram can be evaluated in a straightforward manner yielding

$$W^{[\Gamma]}(P, x, \vec{k}_T, \Delta, N; \eta) = \frac{g_\pi^2 (1-x)\, \mathrm{Tr}\big[(\not{P} - \not{k} + m)\, (\not{k} + \frac{1}{2}\not{\Delta} + m)\, \Gamma\, (\not{k} - \frac{1}{2}\not{\Delta} + m) \big]}{4(2\pi)^3\, (1-\xi^2) P^+ \, D_+ \, D_-} \,, \tag{B.2}$$

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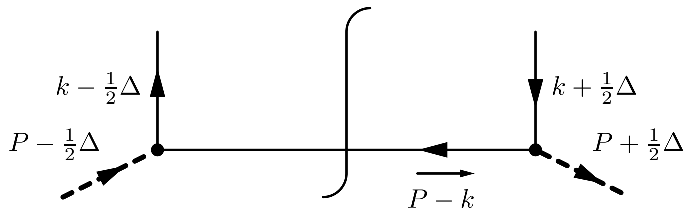

Figure 3: Lowest nontrivial order diagram contributing to the GTMDs in the spectator model for a pion.

where the denominators $D_\pm$ are given by

$$D_\pm = \left(\vec{k}_T \pm \frac{(1-x)}{2(1\mp\xi)}\,\vec{\Delta}_T\right)^2 + m^2 - \frac{(1-x)\,(x\mp\xi)}{(1\mp\xi)^2}\,M^2\,, \tag{B.3}$$

and k^- is fixed by the cut in the diagram to be

$$k^- = \frac{\frac{1}{4}\vec{\Delta}_T^2 + M^2}{2(1-\xi^2)P^+} - \frac{\vec{k}_T^2 + m^2}{2(1-x)P^+}\,. \tag{B.4}$$

Since the calculation is carried out only to lowest order in perturbation theory, no effect due to the Wilson line enters. As a consequence, the trace in (B.2) actually does not depend on the parameter η.

Using now the expression (B.2) and the definitions for the GTMDs in eqs. (3.7)–(3.18) one obtains

$$E_2^e = \frac{4C\,m}{(1-\xi^2)M}\left[\frac{1}{4}\vec{\Delta}_T^2 + \frac{1}{2}(1+\xi^2)M^2\right], \tag{B.5}$$

$$\tilde{E}_2^e = 0\,, \tag{B.6}$$

$$F_1^e = \frac{2C}{1-x}\left[(1-\xi^2)\,(\vec{k}_T^2+m^2) + \xi(1-x)\,\vec{k}_T\cdot\vec{\Delta}_T - \frac{1}{4}(1-x)^2\vec{\Delta}_T^2\right], \tag{B.7}$$

$$F_2^{k,e} = \frac{4C}{1-\xi^2}\left[\frac{1}{4}\vec{\Delta}_T^2 + \frac{1}{2}(1+\xi^2)M^2\right], \tag{B.8}$$

$$\begin{aligned} F_2^{\Delta,e} = \frac{C}{(1-x)(1-\xi^2)}\Bigg[&\xi(1-\xi^2)\,(\vec{k}_T^2+m^2) - (1-x)\,(1-\xi^2)\,\vec{k}_T\cdot\vec{\Delta}_T \\ &-\xi(1-x)^2\left(\frac{1}{4}\vec{\Delta}_T^2 + M^2\right)\Bigg], \end{aligned} \tag{B.9}$$

$$\begin{aligned} F_3^e = -\frac{C}{(1-x)(1-\xi^2)M^2}\Bigg[&\frac{1}{4}(1-\xi^2)\,(\vec{k}_T^2+m^2)\,\vec{\Delta}_T^2 + \xi(1-x)\,\vec{k}_T\cdot\vec{\Delta}_T\left(\frac{1}{4}\vec{\Delta}_T^2+M^2\right) \\ &-(1-x)^2\left(\frac{1}{4}\vec{\Delta}_T^2+M^2\right)^2\Bigg], \end{aligned} \tag{B.10}$$

$$\tilde{G}_1^e = 2C\,M^2\,, \tag{B.11}$$

$$G_2^{k,e} = \frac{4C\,\xi}{1-\xi^2}\left[\frac{1}{4}\vec{\Delta}_T^2 + M^2\right], \tag{B.12}$$

$$G_2^{\Delta,e} = \frac{C}{(1-x)(1-\xi^2)} \left[(1-\xi^2)\,(\vec{k}_T^2 + m^2) - (1-x)^2 \left(\frac{1}{4}\vec{\Delta}_T^2 + M^2 \right) \right] , \tag{B.13}$$

$$\tilde{G}_3^e = -\frac{C}{1-\xi^2} \left[\frac{1}{4}\vec{\Delta}_T^2 + M^2 \right] , \tag{B.14}$$

$$H_1^{k,e} = 0 , \tag{B.15}$$

$$H_1^{\Delta,e} = -2C\,mM , \tag{B.16}$$

$$H_2^e = \frac{4C\,\xi m}{(1-\xi^2)M} \left[\frac{1}{4}\vec{\Delta}_T^2 + M^2 \right] , \tag{B.17}$$

$$\tilde{H}_2^e = 0 , \tag{B.18}$$

$$H_3^{k,e} = 0 , \tag{B.19}$$

$$H_3^{\Delta,e} = \frac{C\,m}{(1-\xi^2)M} \left[\frac{1}{4}\vec{\Delta}_T^2 + M^2 \right] , \tag{B.20}$$

with

$$C = \frac{g_\pi^2 (1-x)}{2(2\pi)^3\,(1-\xi^2)\,D_+\,D_-} . \tag{B.21}$$

To shorten the notation we have suppressed the arguments of the GTMDs. All (naïve) T-odd GTMDs vanish to lowest order in perturbation theory investigated here. To get nonzero results for these functions requires considering at least one-loop corrections that include effects from the Wilson line. On the other hand, the vanishing of the T-even parts of $\tilde{E}_2^e$, $H_1^{k,e}$, $\tilde{H}_2^e$, $H_3^{k,e}$ has to be attributed to the simplicity of the model.

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